With the second term ($n = 1$), $\frac{1}{\pi} \approx \frac{2\sqrt{2}}{9801}\left(\frac{1103}{1} + \frac{4!\,(1103 + 26{,}390)}{396^4}\right) \Rightarrow \pi \approx 3.141\,592\,653\,589\,793\,878$, so we get 15 correct decimal places of π.

39. (a) Since $\sum a_n$ is absolutely convergent, and since $\left|a_n^+\right| \leq |a_n|$ and $\left|a_n^-\right| \leq |a_n|$ (because a_n^+ and a_n^- each equal either a_n or 0), we conclude by the Comparison Test that both $\sum a_n^+$ and $\sum a_n^-$ must be absolutely convergent. *Or:* Use Theorem 12.2.8 [ET 11.2.8].

(b) We will show by contradiction that both $\sum a_n^+$ and $\sum a_n^-$ must diverge. For suppose that $\sum a_n^+$ converged. Then so would $\sum\left(a_n^+ - \frac{1}{2}a_n\right)$ by Theorem 12.2.8 [ET 11.2.8]. But $\sum\left(a_n^+ - \frac{1}{2}a_n\right) = \sum\left[\frac{1}{2}\left(a_n + |a_n|\right) - \frac{1}{2}a_n\right] = \frac{1}{2}\sum|a_n|$, which diverges because $\sum a_n$ is only conditionally convergent. Hence, $\sum a_n^+$ can't converge. Similarly, neither can $\sum a_n^-$.

40. Let $\sum b_n$ be the rearranged series constructed in the hint. [This series can be constructed by virtue of the result of Exercise 39(b).] This series will have partial sums s_n that oscillate in value back and forth across r. Since $\lim\limits_{n\to\infty} a_n = 0$ (by Theorem 12.2.6 [ET 11.2.6]), and since the size of the oscillations $|s_n - r|$ is always less than $|a_n|$ because of the way $\sum b_n$ was constructed, we have that $\sum b_n = \lim\limits_{n\to\infty} s_n = r$.

12.7 Strategy for Testing Series — ET 11.7

1. $\frac{1}{n + 3^n} < \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$ for all $n \geq 1$. $\sum\limits_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$ is a convergent geometric series $\left[|r| = \frac{1}{3} < 1\right]$, so $\sum\limits_{n=1}^{\infty}\frac{1}{n + 3^n}$ converges by the Comparison Test.

2. $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|} = \lim\limits_{n\to\infty}\sqrt[n]{\left|\frac{(2n+1)^n}{n^{2n}}\right|} = \lim\limits_{n\to\infty}\frac{2n+1}{n^2} = \lim\limits_{n\to\infty}\left(\frac{2}{n} + \frac{1}{n^2}\right) = 0 < 1$, so the series $\sum\limits_{n=1}^{\infty}\frac{(2n+1)^n}{n^{2n}}$ converges by the Root Test.

3. $\lim\limits_{n\to\infty}|a_n| = \lim\limits_{n\to\infty}\frac{n}{n+2} = 1$, so $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}(-1)^n\frac{n}{n+2}$ does not exist. Thus, the series $\sum\limits_{n=1}^{\infty}(-1)^n\frac{n}{n+2}$ diverges by the Test for Divergence.

4. $b_n = \frac{n}{n^2+2} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since $\left(\frac{x}{x^2+2}\right)' = \frac{(x^2+2)(1) - x(2x)}{(x^2+2)^2} = \frac{2 - x^2}{(x^2+2)^2} < 0$ for $x \geq \sqrt{2}$. Also, $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty}\frac{n}{n^2+2} = \lim\limits_{n\to\infty}\frac{1/n}{1 + 2/n^2} = 0$. Thus, the series $\sum\limits_{n=1}^{\infty}(-1)^n\frac{n}{n^2+2}$ converges by the Alternating Series Test.

5. $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{(n+1)^2\,2^n}{(-5)^{n+1}}\cdot\frac{(-5)^n}{n^2\,2^{n-1}}\right| = \lim\limits_{n\to\infty}\frac{2(n+1)^2}{5n^2} = \frac{2}{5}\lim\limits_{n\to\infty}\left(1 + \frac{1}{n}\right)^2 = \frac{2}{5}(1) = \frac{2}{5} < 1$, so the series $\sum\limits_{n=1}^{\infty}\frac{n^2\,2^{n-1}}{(-5)^n}$ converges by the Ratio Test.

6. Use the Limit Comparison Test with $a_n = \dfrac{1}{2n+1}$ and $b_n = \dfrac{1}{n}$: $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \lim\limits_{n\to\infty} \dfrac{n}{2n+1} = \lim\limits_{n\to\infty} \dfrac{1}{2+(1/n)} = \dfrac{1}{2} > 0$.

Since the harmonic series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n}$ diverges, so does $\sum\limits_{n=1}^{\infty} \dfrac{1}{2n+1}$. [*Or:* Use the Integral Test.]

7. Let $f(x) = \dfrac{1}{x\sqrt{\ln x}}$. Then f is positive, continuous, and decreasing on $[2,\infty)$, so we can apply the Integral Test.

Since $\displaystyle\int \frac{1}{x\sqrt{\ln x}}\,dx \;\begin{bmatrix} u = \ln x, \\ du = dx/x \end{bmatrix} = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{\ln x} + C$, we find

$\displaystyle\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to\infty} \int_2^t \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to\infty} \left[2\sqrt{\ln x}\right]_2^t = \lim_{t\to\infty} \left(2\sqrt{\ln t} - 2\sqrt{\ln 2}\right) = \infty$. Since the integral diverges, the given series $\sum\limits_{n=2}^{\infty} \dfrac{1}{n\sqrt{\ln n}}$ diverges.

8. $\displaystyle\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!} = \sum_{k=1}^{\infty} \frac{2^k}{(k+1)(k+2)}$. Using the Ratio Test, we get

$\displaystyle\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{2^{k+1}}{(k+2)(k+3)} \cdot \frac{(k+1)(k+2)}{2^k}\right| = \lim_{k\to\infty} \left(2 \cdot \frac{k+1}{k+3}\right) = 2 > 1$, so the series diverges.

Or: Use the Test for Divergence.

9. $\displaystyle\sum_{k=1}^{\infty} k^2 e^{-k} = \sum_{k=1}^{\infty} \frac{k^2}{e^k}$. Using the Ratio Test, we get

$\displaystyle\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{(k+1)^2}{e^{k+1}} \cdot \frac{e^k}{k^2}\right| = \lim_{k\to\infty} \left[\left(\frac{k+1}{k}\right)^2 \cdot \frac{1}{e}\right] = 1^2 \cdot \frac{1}{e} = \frac{1}{e} < 1$, so the series converges.

10. Let $f(x) = x^2 e^{-x^3}$. Then f is continuous and positive on $[1,\infty)$, and $f'(x) = \dfrac{x(2-3x^3)}{e^{x^3}} < 0$ for $x \ge 1$, so f is decreasing on $[1,\infty)$ as well, and we can apply the Integral Test. $\int_1^{\infty} x^2 e^{-x^3}\,dx = \lim\limits_{t\to\infty} \left[-\frac{1}{3}e^{-x^3}\right]_1^t = \frac{1}{3e}$, so the integral converges, and hence, the series converges.

11. $b_n = \dfrac{1}{n\ln n} > 0$ for $n \ge 2$, $\{b_n\}$ is decreasing, and $\lim\limits_{n\to\infty} b_n = 0$, so the given series $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^{n+1}}{n\ln n}$ converges by the Alternating Series Test.

12. The series $\sum\limits_{n=1}^{\infty} \sin n$ diverges by the Test for Divergence since $\lim\limits_{n\to\infty} \sin n$ does not exist.

13. $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{3^{n+1}(n+1)^2}{(n+1)!} \cdot \frac{n!}{3^n n^2}\right| = \lim_{n\to\infty} \frac{3(n+1)^2}{(n+1)n^2} = 3\lim_{n\to\infty} \frac{n+1}{n^2} = 0 < 1$, so the series $\sum\limits_{n=1}^{\infty} \dfrac{3^n n^2}{n!}$ converges by the Ratio Test.

14. $\left|\dfrac{\sin 2n}{1+2^n}\right| \le \dfrac{1}{1+2^n} < \dfrac{1}{2^n} = \left(\dfrac{1}{2}\right)^n$, so the series $\sum\limits_{n=1}^{\infty} \left|\dfrac{\sin 2n}{1+2^n}\right|$ converges by comparison with the geometric series $\sum\limits_{n=1}^{\infty} \left(\dfrac{1}{2}\right)^n$ with $|r| = \frac{1}{2} < 1$. Thus, the series $\sum\limits_{n=1}^{\infty} \dfrac{\sin 2n}{1+2^n}$ converges absolutely, implying convergence.

15. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!}{2\cdot 5\cdot 8\cdot\cdots\cdot(3n+2)[3(n+1)+2]}\cdot\frac{2\cdot 5\cdot 8\cdot\cdots\cdot(3n+2)}{n!}\right| = \lim_{n\to\infty}\frac{n+1}{3n+5} = \frac{1}{3} < 1,$

so the series $\sum_{n=0}^{\infty}\frac{n!}{2\cdot 5\cdot 8\cdot\cdots\cdot(3n+2)}$ converges by the Ratio Test.

16. Using the Limit Comparison Test with $a_n = \frac{n^2+1}{n^3+1}$ and $b_n = \frac{1}{n}$, we have

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\left(\frac{n^2+1}{n^3+1}\cdot\frac{n}{1}\right) = \lim_{n\to\infty}\frac{n^3+n}{n^3+1} = \lim_{n\to\infty}\frac{1+1/n^2}{1+1/n^3} = 1 > 0.$ Since $\sum_{n=1}^{\infty} b_n$ is the divergent harmonic series, $\sum_{n=1}^{\infty} a_n$ is also divergent.

17. $\lim_{n\to\infty} 2^{1/n} = 2^0 = 1$, so $\lim_{n\to\infty}(-1)^n\, 2^{1/n}$ does not exist and the series $\sum_{n=1}^{\infty}(-1)^n 2^{1/n}$ diverges by the Test for Divergence.

18. $b_n = \frac{1}{\sqrt{n}-1}$ for $n \geq 2$. $\{b_n\}$ is a decreasing sequence of positive numbers and $\lim_{n\to\infty} b_n = 0$, so $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}-1}$ converges by the Alternating Series Test.

19. Let $f(x) = \frac{\ln x}{\sqrt{x}}$. Then $f'(x) = \frac{2-\ln x}{2x^{3/2}} < 0$ when $\ln x > 2$ or $x > e^2$, so $\frac{\ln n}{\sqrt{n}}$ is decreasing for $n > e^2$.

By l'Hospital's Rule, $\lim_{n\to\infty}\frac{\ln n}{\sqrt{n}} = \lim_{n\to\infty}\frac{1/n}{1/\left(2\sqrt{n}\right)} = \lim_{n\to\infty}\frac{2}{\sqrt{n}} = 0$, so the series $\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{\sqrt{n}}$ converges by the Alternating Series Test.

20. $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{k+6}{5^{k+1}}\cdot\frac{5^k}{k+5}\right| = \frac{1}{5}\lim_{k\to\infty}\frac{k+6}{k+5} = \frac{1}{5} < 1$, so the series $\sum_{k=1}^{\infty}\frac{k+5}{5^k}$ converges by the Ratio Test.

21. $\sum_{n=1}^{\infty}\frac{(-2)^{2n}}{n^n} = \sum_{n=1}^{\infty}\left(\frac{4}{n}\right)^n$. $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\frac{4}{n} = 0 < 1$, so the given series is absolutely convergent by the Root Test.

22. $\frac{\sqrt{n^2-1}}{n^3+2n^2+5} < \frac{n}{n^3+2n^2+5} < \frac{n}{n^3} = \frac{1}{n^2}$ for $n \geq 1$, so $\sum_{n=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ converges by the Comparison Test with the convergent p-series $\sum_{n=1}^{\infty} 1/n^2$ $[p = 2 > 1]$.

23. Using the Limit Comparison Test with $a_n = \tan\left(\frac{1}{n}\right)$ and $b_n = \frac{1}{n}$, we have

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\tan(1/n)}{1/n} = \lim_{x\to\infty}\frac{\tan(1/x)}{1/x} \overset{H}{=} \lim_{x\to\infty}\frac{\sec^2(1/x)\cdot(-1/x^2)}{-1/x^2} = \lim_{x\to\infty}\sec^2(1/x) = 1^2 = 1 > 0.$ Since $\sum_{n=1}^{\infty} b_n$ is the divergent harmonic series, $\sum_{n=1}^{\infty} a_n$ is also divergent.

24. $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(n\sin\frac{1}{n}\right) = \lim_{n\to\infty}\frac{\sin(1/n)}{1/n} = \lim_{x\to 0^+}\frac{\sin x}{x} = 1 \neq 0$, so the series $\sum_{n=1}^{\infty} n\sin(1/n)$ diverges by the Test for Divergence.

25. Use the Ratio Test. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!}{e^{(n+1)^2}}\cdot\frac{e^{n^2}}{n!}\right| = \lim_{n\to\infty}\frac{(n+1)n!\cdot e^{n^2}}{e^{n^2+2n+1}n!} = \lim_{n\to\infty}\frac{n+1}{e^{2n+1}} = 0 < 1$, so $\sum_{n=1}^{\infty}\frac{n!}{e^{n^2}}$ converges.

26. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\left(\frac{n^2+2n+2}{5^{n+1}}\cdot\frac{5^n}{n^2+1}\right) = \lim_{n\to\infty}\left(\frac{1+2/n+2/n^2}{1+1/n^2}\cdot\frac{1}{5}\right) = \frac{1}{5} < 1$, so $\sum_{n=1}^{\infty}\frac{n^2+1}{5^n}$ converges by the Ratio Test.

27. $\int_2^{\infty}\frac{\ln x}{x^2}dx = \lim_{t\to\infty}\left[-\frac{\ln x}{x}-\frac{1}{x}\right]_1^t$ [using integration by parts] $\overset{\text{H}}{=} 1$. So $\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$ converges by the Integral Test, and since $\frac{k\ln k}{(k+1)^3} < \frac{k\ln k}{k^3} = \frac{\ln k}{k^2}$, the given series $\sum_{k=1}^{\infty}\frac{k\ln k}{(k+1)^3}$ converges by the Comparison Test.

28. Since $\left\{\frac{1}{n}\right\}$ is a decreasing sequence, $e^{1/n} \le e^{1/1} = e$ for all $n \ge 1$, and $\sum_{n=1}^{\infty}\frac{e}{n^2}$ converges ($p = 2 > 1$), so $\sum_{n=1}^{\infty}\frac{e^{1/n}}{n^2}$ converges by the Comparison Test. (Or use the Integral Test.)

29. $\sum_{n=1}^{\infty}a_n = \sum_{n=1}^{\infty}(-1)^n\frac{1}{\cosh n} = \sum_{n=1}^{\infty}(-1)^n b_n$. Now $b_n = \frac{1}{\cosh n} > 0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n = 0$, so the series converges by the Alternating Series Test.

Or: Write $\frac{1}{\cosh n} = \frac{2}{e^n + e^{-n}} < \frac{2}{e^n}$ and $\sum_{n=1}^{\infty}\frac{1}{e^n}$ is a convergent geometric series, so $\sum_{n=1}^{\infty}\frac{1}{\cosh n}$ is convergent by the Comparison Test. So $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\cosh n}$ is absolutely convergent and therefore convergent.

30. Let $f(x) = \frac{\sqrt{x}}{x+5}$. Then $f(x)$ is continuous and positive on $[1,\infty)$, and since $f'(x) = \frac{5-x}{2\sqrt{x}\,(x+5)^2} < 0$ for $x > 5$, $f(x)$ is eventually decreasing, so we can use the Alternating Series Test. $\lim_{n\to\infty}\frac{\sqrt{n}}{n+5} = \lim_{n\to\infty}\frac{1}{n^{1/2}+5n^{-1/2}} = 0$, so the series $\sum_{j=1}^{\infty}(-1)^j\frac{\sqrt{j}}{j+5}$ converges.

31. $\lim_{k\to\infty}a_k = \lim_{k\to\infty}\frac{5^k}{3^k+4^k} =$ [divide by 4^k] $\lim_{k\to\infty}\frac{(5/4)^k}{(3/4)^k+1} = \infty$ since $\lim_{k\to\infty}\left(\frac{3}{4}\right)^k = 0$ and $\lim_{k\to\infty}\left(\frac{5}{4}\right)^k = \infty$. Thus, $\sum_{k=1}^{\infty}\frac{5^k}{3^k+4^k}$ diverges by the Test for Divergence.

32. $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\sqrt[n]{\left|\frac{(n!)^n}{n^{4n}}\right|} = \lim_{n\to\infty}\frac{n!}{n^4} = \lim_{n\to\infty}\left[\frac{n}{n}\cdot\frac{n-1}{n}\cdot\frac{n-2}{n}\cdot\frac{n-3}{n}\cdot(n-4)!\right]$

$$= \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right)(n-4)!\right] = \infty,$$

so the series $\sum_{n=1}^{\infty}\frac{(n!)^n}{n^{4n}}$ diverges by the Root Test.

33. Let $a_n = \frac{\sin(1/n)}{\sqrt{n}}$ and $b_n = \frac{1}{n\sqrt{n}}$. Then $\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\sin(1/n)}{1/n} = 1 > 0$, so $\sum_{n=1}^{\infty}\frac{\sin(1/n)}{\sqrt{n}}$ converges by limit comparison with the convergent p-series $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ $[p = 3/2 > 1]$.

34. $0 \le n\cos^2 n \le n$, so $\dfrac{1}{n + n\cos^2 n} \ge \dfrac{1}{n+n} = \dfrac{1}{2n}$. Thus, $\sum\limits_{n=1}^{\infty} \dfrac{1}{n + n\cos^2 n}$ diverges by comparison with $\sum\limits_{n=1}^{\infty} \dfrac{1}{2n}$, which is a constant multiple of the (divergent) harmonic series.

35. $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = \lim\limits_{n\to\infty} \left(\dfrac{n}{n+1}\right)^{n^2/n} = \lim\limits_{n\to\infty} \dfrac{1}{[(n+1)/n]^n} = \dfrac{1}{\lim\limits_{n\to\infty}(1+1/n)^n} = \dfrac{1}{e} < 1$, so the series $\sum\limits_{n=1}^{\infty} \left(\dfrac{n}{n+1}\right)^{n^2}$ converges by the Root Test.

36. Note that $(\ln n)^{\ln n} = \left(e^{\ln \ln n}\right)^{\ln n} = \left(e^{\ln n}\right)^{\ln \ln n} = n^{\ln \ln n}$ and $\ln \ln n \to \infty$ as $n \to \infty$, so $\ln \ln n > 2$ for sufficiently large n. For these n we have $(\ln n)^{\ln n} > n^2$, so $\dfrac{1}{(\ln n)^{\ln n}} < \dfrac{1}{n^2}$. Since $\sum\limits_{n=2}^{\infty} \dfrac{1}{n^2}$ converges $[p = 2 > 1]$, so does $\sum\limits_{n=2}^{\infty} \dfrac{1}{(\ln n)^{\ln n}}$ by the Comparison Test.

37. $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = \lim\limits_{n\to\infty} (2^{1/n} - 1) = 1 - 1 = 0 < 1$, so the series $\sum\limits_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)^n$ converges by the Root Test.

38. Use the Limit Comparison Test with $a_n = \sqrt[n]{2} - 1$ and $b_n = 1/n$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2^{1/n} - 1}{1/n} = \lim_{x\to\infty} \frac{2^{1/x} - 1}{1/x} \overset{H}{=} \lim_{x\to\infty} \frac{2^{1/x} \cdot \ln 2 \cdot (-1/x^2)}{-1/x^2} = \lim_{x\to\infty} (2^{1/x} \cdot \ln 2) = 1 \cdot \ln 2 = \ln 2 > 0.$$

So since $\sum\limits_{n=1}^{\infty} b_n$ diverges (harmonic series), so does $\sum\limits_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)$.

Alternate solution: $\sqrt[n]{2} - 1 = \dfrac{1}{2^{(n-1)/n} + 2^{(n-2)/n} + 2^{(n-3)/n} + \cdots + 2^{1/n} + 1}$ [rationalize the numerator] $\ge \dfrac{1}{2n}$, and since $\sum\limits_{n=1}^{\infty} \dfrac{1}{2n} = \dfrac{1}{2} \sum\limits_{n=1}^{\infty} \dfrac{1}{n}$ diverges (harmonic series), so does $\sum\limits_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)$ by the Comparison Test.

12.8 Power Series — ET 11.8

1. A power series is a series of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$, where x is a variable and the c_n's are constants called the coefficients of the series.

   More generally, a series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$ is called a power series in $(x-a)$ or a power series centered at a or a power series about a, where a is a constant.

2. (a) Given the power series $\sum_{n=0}^{\infty} c_n (x-a)^n$, the radius of convergence is:

   (i) 0 if the series converges only when $x = a$

   (ii) ∞ if the series converges for all x, or

   (iii) a positive number R such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$.

   In most cases, R can be found by using the Ratio Test.

   (b) The interval of convergence of a power series is the interval that consists of all values of x for which the series converges. Corresponding to the cases in part (a), the interval of convergence is: (i) the single point $\{a\}$, (ii) all real numbers; that is, the real number line $(-\infty, \infty)$, or (iii) an interval with endpoints $a - R$ and $a + R$ which can contain neither, either, or both of the endpoints. In this case, we must test the series for convergence at each endpoint to determine the interval of convergence.