Set operations: Union and Intersection

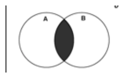
Given two sets A and B

- The **union** of A and B is defined as the set

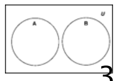
  $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

- The **intersection** of X and Y is defined as the se

  $A \cap B = \{ x \mid x \in A \text{ and } x \in B\}$



  - Two sets A and B are disjoint if $A \cap B = \varnothing$



Set operations: Union & Intersection

- examples
  - {1, 2, 3} U {3, 4, 5} = {1, 2, 3, 4, 5}
  - {New York, Washington} U {3, 4} = {New York, Washington, 3, 4}
  - {1, 2} U ∅ = {1, 2}
  - {1, 2, 3} ∩ {3, 4, 5} = {3}
  - {1, 2, 3} and {3, 4, 5} are not disjoint
  - {New York, Washington} ∩ {3, 4} = ∅
    - No elements in common, these sets are disjoint
  - {1, 2} ∩ ∅ = ∅
    - Any set intersection with the empty set yields the empty set

Set Operations: Relative Complement

- The relative complement (difference) of two sets is the set defined as:

  $A - B = \{ x \mid x \in A \text{ and } x \notin B \}$

- Given: A = {a, b, c, d} and B = {a, c, f, g}

  A — B = {b, d}

  B — A = {f, g}

Note: The complement of a set A contained in a Universal set $\mathscr{U}$ is the set A′ = $\mathscr{U}$ – A

Set operations: relative Complement

- Examples
  - {1, 2, 3} - {3, 4, 5} = {1, 2}
  - {New York, Washington} - {3, 4} = {New York, Washington}
  - {1, 2} - Ø = {1, 2}

Set operations: Complement

Now, assuming U = Z

- Complement of {1, 2, 3} = { ..., -2, -1, 0, 4, 5, 6, ... }

Example

- If X={1, 4, 7, 10}, Y={1, 2, 3, 4, 5}, U={1, 2, 3, ..., 10}
  - X U Y =
  - X ∩ Y =
  - X – Y =
  - Y – X =
  - X' =
  - Y' =
  - X'' =

Example

- If X={1, 4, 7, 10}, Y={1, 2, 3, 4, 5}, U={1, 2, 3, ..., 10}
  - X U Y = {1, 2, 3, 4, 5, 7, 10}
  - X ∩ Y = {1, 4}
  - X – Y = {7, 10}
  - Y – X = {2, 3, 5}
  - X' = {2, 3, 5, 6, 8, 9}
  - Y' = {6, 7 ,8 , 9, 10}
  - X'' = {1, 4, 7, 10}

Properties of Set operations

- Properties of the union operation

A U ∅ = A	Identity law
A U U = U	Domination law
A U A = A	Idempotent law
A U B = B U A	Commutative law
A U (B U C) = (A U B) U C	Associative law

- Properties of the intersection operation

A ∩ U = A	Identity law
A ∩ ∅ = ∅	Domination law
A ∩ A = A	Idempotent law
A ∩ B = B ∩ A	Commutative law
A ∩ (B ∩ C) = (A ∩ B) ∩ C	Associative law

Properties contd...

- Properties of complement sets

A'' = A	Complementation law
A' U A = U	Complement law
A' ∩ A = ∅	Complement law

Properties of set operations

- Theorem 2.1.10: Let U be a universal set, and A, B and C subsets of U. The following properties hold:

a) Associativity: (A U B) U C = A U (B U C)

(A ∩ B) ∩ C = A ∩ (B ∩ C)

b) Commutativity: A U B = B U A

c) Distributive laws:

A U (B∩C) = (AUB) ∩ (AUC)

A ∩ (BUC) = (A∩B) U (A∩C)

d) Identity laws:

A∩U=A A U∅ = A

Properties of set operations

e) Complement laws:

A U A' = U A∩A' = ⊠

f) Idempotent laws:

AUA = A A∩A = A

g) Bound or Identity laws:

A U 𝒰 = 𝒰 A∩∅ = ∅

h) Absorption laws:

A U (A∩B) = A A ∩ (A U B) = A

Properties of set operations

i) Involution law:

(A')' = A

j) Compliment laws:

∅' = U

𝒰' = ∅

k) De Morgan's laws for sets:

(A U B)' = A' ∩ B'

(A ∩ B)' = A' U B'

Counting Principle:

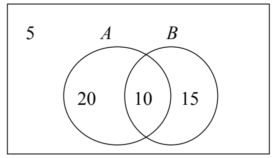
a) If A and B are finite then $A \cup B$ and $A \cap B$ are finite and $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

b) If A and B are disjoint finite sets, then $A \cup B$ is finite and $n(A \cup B) = n(A) + n(B)$

c) Similarly If A, B, C are finite sets then $A \cup B \cup C$ is finite and $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(A \cap C) + n(A \cap B \cap C)$.

Examlples:

1. In a class of 50 college freshmen, 30 are studying JAVA, 25 studying UNIX, and 10 are studying both. How many freshmen are studying either computer language?

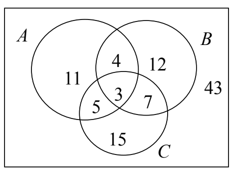


$|A \cup B| = |A| + |B| - |A \cap B|$

Examples:

2. Defect types of an AND gate:

D_1: first input stuck at 0
D_2: second input stuck at 0
D_3: output stuck at 1



Given 100 samples
set *A*: with D_1
set *B*: with D_2
set *C*: with D_3
We have

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

with $|A|=23$, $|B|=26$, $|C|=30$,
$|A \cap B| = 7, |A \cap C| = 8, |B \cap C| = 10,$
$|A \cap B \cap C| = 3$, how many samples have defects? **Ans:57**