SET

เซต

เซต วิธีการเขียนเซต เซตที่เท่ากัน สับเซต เพาเวอร์เซต การดำเนินการของเซต แผนภาพเวนน์-ออยเลอร์ และการแก้โจทย์ปัญหา

Definition

- A set is a group of "objects"
  - People in a class: { Atom, Dos, Dada, View }
  - Courses offered by a department: { MA30101, MA30102... }
  - Colors of a rainbow: { violet, indigo, blue, green, yellow, orange, red}
  - States of matter { solid, liquid, gas, plasma }
  - States in the US: { Alabama, Alaska, Virginia, ... }
  - Sets can contain non-related elements: { 3, a, red, Virginia }

Definition

- Although a set can contain (almost) anything, we will most often use sets of numbers
  - All positive numbers less than or equal to 5: {1, 2, 3, 4, 5}
  - A few selected real numbers: { 2.1, π, 0, -6.32, e }
- Order does not matter
  - {1, 2, 3, 4, 5} is equivalent to {3, 5, 2, 4, 1}

Properties

- Sets do not have duplicate elements
  - Consider the set of vowels in the alphabet.
    - It makes no sense to list them as {a, a, a, e, i, o, o, o, o, o, u}
    - What we really want is just {a, e, i, o, u}
- Sets can contain other sets
  - S = { {1}, {2}, {3} }
  - T = { {1}, {{2}}, {{{3}}} }
  - V = { {{1}, {{2}}}, {{{3}}}, { {1}, {{2}}, {{{3}}} } }

Note that $1 \neq \{1\} \neq \{\{1\}\} \neq \{\{\{1\}\}\}$
They are all different

Specifications

- Sets are usually represented by a capital letter (A, B, S, etc.)
- Elements are usually represented by an italic lower-case letter (a, x, y, etc.)
- Easiest way to specify a set is to list all the elements: A = {1, 2, 3, 4, 5}
  - Not always feasible for large or infinite sets

Specifications

- A set is said to "contain" the various "members" or "elements" that make up the set
  - If an element a is a member of (or an element of) a set S, we use then notation $a \in S$
    - $4 \in \{1, 2, 3, 4\}$
  - If an element is not a member of (or an element of) a set S, we use the notation a S
    - $7 \notin \{1, 2, 3, 4\}$
    - Atom $\notin$ {1, 2, 3, 4}

Specifications

- Can use an ellipsis (…): B = {0, 1, 2, 3, …}
  - Consider the set C = {3, 5, 7, …}. What comes next?
    - If the set is all odd integers greater than 2, it is 9
    - If the set is all prime numbers greater than 2, it is 11
- Can use set-builder notation
  - D = {x | x is prime and x > 2}
  - E = {x | x is odd and x > 2}
  - The vertical bar means "such that"
  - How to read these sets ?

Finite and infinite sets

- Finite sets
  - Examples:
    - A = {1, 2, 3, 4}
    - B = {x | x is an integer, 1 < x < 4}
- Infinite sets
  - Examples:
    - Z = {integers} = {…, -3, -2, -1, 0, 1, 2, 3,…}
    - S = {x| x is a real number and 1 < x < 4} = (1, 4)

Often used sets

- N = {1, 2, 3, …} is the set of natural numbers
- Z = {…, -2, -1, 0, 1, 2, …} is the set of integers (Z or I)
- Z+ = {1, 2, 3, …} is the set of positive integers (whole numbers)
  - Note that people disagree on the exact definitions of whole numbers and natural numbers
- Q = {p/q | p ε Z, q ≠ 0} is the set of rational numbers
  - Any number that can be expressed as a fraction of two integers (where the bottom one is not zero)
  - Q* is the set of nonzero rational numbers

Often used sets

- R is the set of real numbers
- R+ is the set of positive real numbers
- R* is the set of nonzero real numbers
- C is the set of complex numbers

Some important sets

- $\mathscr{U}$ is the universal set – the set of all of elements (or the "universe") from which given any set is drawn
  - For the set {-2, 0.4, 2}, $\mathscr{U}$ would be the real numbers
  - For the set {0, 1, 2}, $\mathscr{U}$ could be the natural numbers (zero and up), the integers, the rational numbers, or the real numbers, depending on the context
- Universal set: the set of all elements about which we make assertions.
- The empty set ∅ = { } has no elements.
  - Also called null set or void set.

More Examples

- For the set of the students in this class, $\mathscr{U}$ would be all the students in the University (or perhaps all the people in the world)
- For the set of the vowels of the alphabet, $\mathscr{U}$ would be all the letters of the alphabet
- To differentiate $\mathscr{U}$ from $\mathscr{U}$ (which is a set operation), the universal set is written in a different font (and in bold and italics)

The empty set 1

- If a set has zero elements, it is called the empty (or null) set
  - Written using the symbol ∅
  - Thus, ∅ = { }   ← VERY IMPORTANT
  - If you get confused about the empty set in a problem, try replacing ∅ by { }
- As the empty set is a set, it can be a element of other sets
  - {∅, 1, 2, 3, x } is a valid set

The empty set 2

- Note that ∅ ≠ {∅}
  - The first is a set of zero elements
  - The second is a set of 1 element (that one element being the empty set)
- Replace ∅ by { }, and you get: { } ≠ { { } }
  - It's easier to see that they are not equal that way

Cardinality

- Cardinality of a set A (in symbols |A|) is the number of elements in A
- Examples:
  - If A = {1, 2, 3} then |A| = 3
  - If B = {x | x is a natural number and 1< x< 9}
  - then |B| = 9
- Infinite cardinality
  - Countable (e.g., natural numbers, integers)
  - Uncountable (e.g., real numbers)

Set equality

- Two sets are equal if they have the same elements
  - {1, 2, 3, 4, 5} = {5, 4, 3, 2, 1}
    - Remember that order does not matter!
  - {1, 2, 3, 2, 4, 3, 2, 1} = {4, 3, 2, 1}
    - Remember that duplicate elements do not matter!
- Two sets are not equal if they do not have the same elements
  - {1, 2, 3, 4, 5} ≠ {1, 2, 3, 4}

Subsets

- If all the elements of a set S are also elements of a set T, then S is a subset of T
  - For example, if S = {2, 4, 6} and T = {1, 2, 3, 4, 5, 6, 7}, then S is a subset of T
  - This is specified by S ⊆T
    - Or by {2, 4, 6} ⊂ {1, 2, 3, 4, 5, 6, 7}
- If S is not a subset of T, it is written as such: S ⊈ T
  - For example, {1, 2, 8} ⊈ {1, 2, 3, 4, 5, 6, 7}

Subsets

- Note that any set is a subset of itself!
  - Given set S = {2, 4, 6}, since all the elements of S are elements of S, S is a subset of itself
  - This is kind of like saying 5 is less than or equal to 5
  - Thus, for any set S, S ⊆ S

Subsets

- The empty set is a subset of all sets (including itself!)
  - Recall that all sets are subsets of themselves
- All sets are subsets of the universal set
- The textbook has this gem to define a subset:
  - Forall x ($x \in A \rightarrow x \in B$)
  - English translation: for all possible elements of a set, if x is an element of A, then x is an element of B
  - This type of notation will be gone over later

Proper Subsets

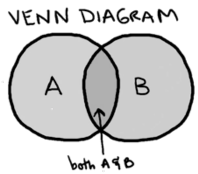
- If S is a subset of T, and S is not equal to T, then S is a proper subset of T
  - Let T = {0, 1, 2, 3, 4, 5}
  - If S = {1, 2, 3}, S is not equal to T, and S is a subset of T
  - A proper subset is written as $S \subset T$
  - Let R = {0, 1, 2, 3, 4, 5}. R is equal to T, and thus is a subset (but not a proper subset) or T
    - Can be written as: $R \subseteq T$ and $R \not\subset T$ (or just R = T)
  - Let Q = {4, 5, 6}. Q is neither a subset of T nor a proper subset of T

Proper Subsets

- The difference between “subset” and “proper subset” is like the difference between “less than or equal to” and “less than” for numbers
- For a given set X, The empty set and the set X, itself are the only, improper subset of set X.

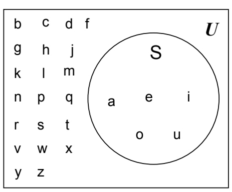
Venn diagrams

- A Venn diagram provides a graphic view of sets
- Set union, intersection, difference, symmetric difference and complements can be easily and visually identified



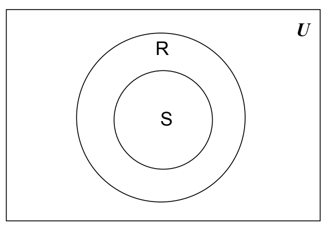
Venn diagrams

- Represents sets graphically
  - The box represents the universal set
  - Circles represent the set(S)
- Consider set S, which is the set of all vowels in the alphabet
- The individual elements are usually not written in a Venn diagram



Proper subsets: Venn diagram

$S \subset R$



Subsets: review

- X is a subset of Y if every element of X is also contained in Y (in symbols X ⊆ Y)
- Equality: X = Y if X ⊆ Y and Y ⊆ X, i.e., X = Y
  whenever x ∈ X, then x ∈ Y, and
  whenever x ∈ Y, then x ∈ X
- X is a proper subset of Y if X ⊆ Y but Y ⊈ X
  - Observation: ∅ is a subset of every set

Question

- What is the meaning of X ⊆ Y and X≠ Y ?

Question

- What is the meaning of X ⊆ Y and X≠ Y ?
- X is a proper subset of Y

Power sets

- Given the set S = {0, 1}. What are all the possible subsets of S?
  - They are: ∅ (as it is a subset of all sets), {0}, {1}, and {0, 1}
  - The power set of S (written as P(S)) is the set of all the subsets of S
  - P(S) = {∅, {0}, {1}, {0,1} }
    - Note that |S| = 2 and |P(S)| = 4

Power sets

- Let T = {0, 1, 2}. The P(T) = {∅, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2} }
  - Note that |T| = 3 and |P(T)| = 8
- P(∅) = { ∅ }
  - Note that |∅| = 0 and |P(∅)| = 1
- If a set has n elements, then the power set will have 2^n elements

If $|A|=n$, then $|P(A)|=2^n$.

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}=2^n, \text{ for } n \geq 0$$